

Substantial Change: Continuous, Consistent, Objective

In his book *Invariances*, Robert Nozick rehearses an old problem about the nature of change in continuous time:

If time is continuous (or even if it is dense), then between any two instants there are an infinite number of other instants.... So with a changing object in continuous time, either there will not be a last instant when it is in one state or there will not be a first instant when it is in the other.... Yet there seems to be no special reason to describe the change in one of these ways rather than the other.... [I]t seems arbitrary to choose one of the two alternative ways to knit together intervals that are exhaustive yet not overlapping.¹

Although he gives no indication that he is aware of the problem’s ancient pedigree,² Nozick does seem to appreciate the difficulty of solving it. Indeed, he goes so far as to suggest that solving the problem requires us to accept that the world contains ubiquitous (if short-lived) situations that are contradictory:

Therefore, it might be suggested that when an object changes its state, there is *both* a last instant of one state *and* a first instant of the other.... There would be an instant when the object is...both red and not red. And, in the case of the object that ceases

¹ Robert Nozick, *Invariances: The Structure of the Objective World* (Cambridge, MA: Harvard University Press, 2001), 303–04.

² The problem receives a brief discussion in Plato, *Parmenides* 156c–157a, and an extensive treatment in Aristotle, *Physics* VI. See Richard Sorabji, “Aristotle on the Instant of Change,” *Proceedings of the Aristotelian Society*, supp. vol. 50: 1 (1976): 69–89.

to exist, there would be an instant when the object both exists and does not exist. These contradictions hold only for an instant, though; it is no wonder that we do not notice them.³

Nozick cautions against exaggerating the cost of accepting “such delimited and motivated exceptions to the principle of noncontradiction”: “Logic would not crumble,” he says; “Reason would not totter.”⁴ However, in classical logic even one true contradiction implies the truth of *every* contradiction, and the truth of every contradiction is clearly an intolerable cost. Avoiding this cost requires either refusing to accept even one true contradiction or else abandoning classical logic in favor of a “paraconsistent” logic that rejects such intuitively valid inference forms as disjunctive syllogism: $\varphi \vee \psi, \neg \varphi \vdash \psi$.

Fortunately, we can solve a particularly important version of the problem while assuming that time is continuous and that the laws of classical logic hold without exception. Neither of those assumptions should be abandoned lightly. First, much of current physics presupposes the continuity of time, and nothing in current physics rules it out. Granted, some physicists conjecture that time “becomes discrete” (that is, the discreteness of time is revealed) at the Planck scale, on the order of 10^{-43} seconds. But these same physicists admit that they are at least two dozen orders of magnitude away from measuring intervals that brief, that they have no confident idea about the nature of time inside such intervals, and that time looks continuous at every scale they *have* been

³ Nozick, *Invariances*, *op. cit.*, 303. Such inconsistency is embraced more enthusiastically in, for example, Graham Priest, “To Be and Not to Be: Dialectical Tense Logic,” *Studia Logica* 41: 2–3 (1982): 249–68.

⁴ Nozick, *Invariances*, *op. cit.*, 304.

able to measure.⁵ Second, even the critics of classical logic say that we should operate with it except when we have compelling reasons not to. Here we do not, as I will argue.

These assumptions imply the following consequences for any *substantial change*, that is, any case in which a metaphysical substance comes into or goes out of existence. Necessarily, for any substance that comes into existence, there is either a last instant at which the substance does not yet exist or a first instant at which it exists, but not both. For any substance that goes out of existence, there is either a last instant at which the substance still exists or a first instant at which it no longer exists, but not both. I did not always exist; I came into existence. So there was a last instant at which I did not yet exist or a first instant at which I did, but not both. Which *was* it—which disjunct in that exclusive disjunction is true? Inquiring minds want to know!

Over the centuries, the general topic of the “instant of change” has spawned an intricate variety of problems and proposed solutions.⁶ But arguably the most serious problem in this area is the one I just raised for substantial change, because substantial change seems more clearly objective—less up to us, more up to the world apart from us—than qualitative change. Questions about the instant at which, say, a substance changes color depend for their answers on how we conceive of color in general and of particular colors, our sensitivity to color, and so on. Is a substance still partly red if the extent of redness on its surface falls below what (any or most?) observers could possibly detect? Or does the notion of an undetectably red object make no sense? Questions about a substance’s transition from rest to motion depend for their answers on idealizing away various facts about the actual world, as I discuss below. But it is up to the world apart from

⁵ See Lisa Randall, *Knocking on Heaven’s Door: How Physics and Scientific Thinking Illuminate the Universe and the Modern World* (New York: Harper Collins, 2011), 411.

⁶ Niko Strobach, *The Moment of Change: A Systematic History in the Philosophy of Space and Time* (Dordrecht, Netherlands: Kluwer Academic Publishers, 1998), provides a comprehensive historical treatment of the topic.

us—anyway, the world apart from *me*—whether I came into existence or exactly when I did, and what goes for me in that regard goes for everything.

A particular motivation to solve the problem of substantial change ought to be felt by anyone who accepts, as I do, commonsense realism about substances: the view that humans, trees, tables, etc., are as objectively real as anything in the universe and no less real than the parts or the matter that composes them. Substances are not fictions that we concoct out of what there *really* is; at a minimum, we cannot concoct *ourselves* out of something more real. So it is not up to us to *decide* on a solution to the problem of substantial change; it is up to us to discover what the solution objectively is.

I. Idealized Motion

Because my argument will take seriously the metaphor of “moving through time,” let us begin by considering the local (that is, frame-relative) motion of some ordinary substance in continuous time. Imagine a billiard ball held in place at the top of an inclined plane, then released, and then rolling down the plane. Does the ball have a last instant of non-motion or a first instant of motion? Making this question tractable requires some idealization. Unless the ball is perfectly rigid, some of its parts may start to move while other parts remain motionless, making it unclear whether we should classify the ball itself as moving or not. So we might consider, instead, the motion of the ball’s center of mass. But at any temperature above absolute zero, the molecules that compose any material substance are always moving, making it likely, or at least possible, that the substance’s center of mass is also always moving.⁷ To avoid these complications, let us consider, rather than

⁷ Sorabji acknowledges this fact in “Aristotle on the Instant of Change,” *op. cit.*, 88 n. 2, but chooses to ignore it for simplicity’s sake. Even at absolute zero, individual atoms continue to

the motion of the center of mass of some extended substance, the motion of only an unextended point mass: call it ' p '. As far as I can see, none of these idealizations affect the validity of my argument.

I take it as a conceptual truth that if p has a first instant *of* motion, then p is moving— p is in motion—at that instant. Indeed, it seems conceptually true that p is moving at *every* instant of its motion. My proposal, then, is to interpret the present participle in the description “moving at an instant” quite literally, as signaling a *process*. The synonymous description “being in motion at an instant” also strongly suggests a process, in notable contrast to which “being in non-motion at an instant” does not suggest a process. Granted, the description “being *at rest* at an instant” may suggest a process, but only if we construe it as *resting* (recovering, catching one’s breath) at an instant. The logically weaker description “being in non-motion” does not imply resting in that sense, and it carries no suggestion of a process. It would be awkward or else deliberately droll to describe p , even when at rest, as “in the process of not moving.”

If, at any instant t of its motion, p is in the process of moving, then p must have been moving “just before” t and must continue to move “just after” t . More precisely:

- (M) Necessarily, for any p , if p is in motion at t , then p is in motion at any earlier instant that is arbitrarily close to t and in motion at any instant later than t that is arbitrarily close to t ,

“vibrate” according to the laws of quantum mechanics, with what effect on a substance’s center of mass I do not know.

where “arbitrarily close to t ” means “as close to, yet distinct from, t as anyone could specify.”⁸ Condition M holds because it seems false to say that p is moving at t if either (i) p never occupies, at earlier instants arbitrarily close to t , any location different from its location at t , or (ii) p never occupies, at later instants arbitrarily close to t , any location different from its location at t . For (i) describes something that, at t , is *not yet* moving, and (ii) describes something that, at t , is *no longer* moving if it ever was. Neither of those is something that *is* moving at t . Indeed, M gains support from standard physics, which defines p ’s rate of motion—its speed—at t as the first derivative at t of the function of p ’s location over time. If either (i) or (ii), then p ’s rate of motion at t is undefined—there is no such thing as the speed of p —because at t the first derivative is undefined. In that case, surely, p is not moving at t .

Now, let t_E be any of the earlier instants referred to in M. Because p was moving at t_E , it follows from M that p was moving at still earlier instants. Let t_L be any of the later instants referred to in M. Because p will be moving at t_L , it follows from M that p will be moving at still later instants. So, necessarily, no instant is the first or the last instant of p ’s motion. Given the continuity of time, therefore, some instant must have been the last instant of p ’s earlier state of non-motion, and if p ever comes to rest, some instant will be the first instant of p ’s later state of non-motion. Again, because non-motion, as such, is not a process, we face no pressure to say that, for any instant t at which p is in non-motion, p must be in non-motion at earlier and later instants that are arbitrarily close to t .

Hence we have found what Nozick, in my opening quotation, seems to despair of finding: a “special,” objectively defensible reason for saying that neither a first nor a last instant of motion

⁸ See also Norman Kretzmann, “Incipit/Desinit,” in Peter K. Machamer and Roger G. Turnbull, eds., *Motion and Time, Space and Matter* (Columbus: Ohio State University Press, 1976), 101–36, at 103; and Sorabji, “Aristotle on the Instant of Change,” *op. cit.*, 72.

exists and that both a first and a last instant of non-motion exist.⁹ I hasten to emphasize that M is only a necessary condition for p 's being in motion at t , not a definition or a condition that is both necessary and sufficient. Thus nothing I have said conflicts with the assertion from standard physics that, subject to the usual idealization, a baseball thrown straight upward is in non-motion at the apex of its trajectory, before it begins to fall, even though the apex is sandwiched between both earlier and later instants of motion that are arbitrarily close to the apex.¹⁰ M does not imply that the baseball is moving at its apex.

As I conceded earlier, one might use the phrase “being at rest at t ” while intending it in a misleadingly narrow way, that is, as “in the process of resting at t .” I say “misleadingly” because someone who, in the context of discussing motion, uses “at rest” to mean literally “resting” violates the Gricean conversation rule that requires being as informative as you can be in the circumstances without going overboard. If you mean “resting” or “recovering,” rather than simply “not moving,” then you ought to say so. Because anything’s resting is a process, my reasoning implies that there is no first or last instant of that process. Therefore, no substance’s states of moving and resting are contiguous: those states are always separated by at least one instant that belongs to neither state (and if by more than one instant, then of course by infinitely many). Furthermore, “moving” and “resting” are logically contrary descriptions both of which may fail to apply to a given substance at a given instant: for example, the baseball is neither moving nor resting, but simply at rest, at the apex of its trajectory. These consequences of M are therefore welcome.

⁹ On the importance of finding a non-arbitrary answer to the question of motion, see Chris Mortensen, “The Limits of Change,” *Australasian Journal of Philosophy* 63: 1 (1985): 1–10; and Frank Jackson and Robert Pargetter, “A Question about Rest and Motion,” *Philosophical Studies* 53: 1 (1988): 141–46. For a position more tolerant of arbitrariness, see Joseph Wayne Smith, “Time, Change and Contradiction,” *Australasian Journal of Philosophy* 68: 2 (1990): 178–88.

¹⁰ See Sorabji, “Aristotle on the Instant of Change,” *op. cit.*, 73, 86.

II. Application to Substantial Change

Now to apply the analysis of motion to solve the problem of substantial change. The key premise of my solution is that, necessarily, at any instant at which any substance exists the substance is in the process of getting older: in that sense, moving through time.¹¹ Equivalently, and again necessarily, any substance is in the process of *aging* at every instant at which it exists, provided we understand aging as merely getting older, regardless of whether the process changes the substance intrinsically. If, at any instant t of its existence, the substance is in the process of aging, then it must have been aging “just before” t and must continue to age “just after” t . More precisely:

- (A) Necessarily, any substance that is aging at t is aging at earlier instants that are arbitrarily close to t and aging at instants later than t that are arbitrarily close to t ,

with “arbitrarily close” understood as in M, above.

Condition A holds because it seems false to say that a substance is aging at t if either (iii) the substance never was younger than it is at t , or (iv) the substance never will be older than it is at t . For (iii) describes a substance that, at t , is *not yet* moving through time, and (iv) describes a substance that, at t , is *no longer* moving through time if it ever was. Neither of those is a substance that *is* moving through time at t . We can press the analogy a bit further. A substance’s rate of aging at t (such as one second per second) will be the first derivative at t of the function of the substance’s age over time. If either (iii) or (iv), then the substance’s rate of aging at t is undefined—there is no such thing as the substance’s rate of aging—because at t the first derivative is undefined. In that case, surely, the substance is not aging at t .

¹¹ This premise is compatible with the existence of an atemporal, literally timeless substance, which some theists take God to be. In such a case, the premise comes out vacuously true, because no timeless substance exists *at* any instant of time.

Now, let t_E be any of the earlier instants referred to in A. Because the substance was aging at t_E , there must be still earlier instants at which it was aging. Let t_L be any of the later instants referred to in A. Because the substance will be aging at t_L , there must be still later instants at which it will be aging. It follows that, necessarily, no instant is the first or the last instant of any substance's aging or, therefore, the first or the last instant of any substance's existence. It also follows that, necessarily, no substance exists for only one instant, another welcome result.¹² Given the continuity of time, then, for any substance that comes into existence there is a last instant at which the substance does not yet exist, and for any substance that goes out of existence there is a first instant at which the substance no longer exists.¹³

The attentive reader will notice that conditions M and A are plausible *only* if time is non-discrete.¹⁴ For if time is discrete, then M implies the falsehood that any object that is moving at any time is moving at literally all times, and A implies the falsehood that any substance that is aging at any time is aging at literally all times. One might respond to this implication, boldly, by insisting that M and A show that our concepts of motion and aging *require* non-discrete time or, more cautiously, by offering M and A as (partial) conceptual analyses of motion and aging only *in* non-discrete time. Because the problem I address in this paper arises in the first place only if time is non-discrete, and because time is to all appearances non-discrete anyway, I make only the more cautious response.

¹² My view shares this result with asymmetric views that rule out only the first or only the last instant of a substance's existence, and I think it is better motivated than those asymmetric views. Furthermore, I cannot see what explanatory need there is for instantaneous substances or what problems they would solve.

¹³ Because, obviously, any substance moves through time only if the substance exists, we face no pressure to treat *non-existence* as a process.

¹⁴ I thank an anonymous referee from this JOURNAL for asking about both this issue and the issue of endurantism, discussed below in section IV.

III. No Contiguous Succession of Substances

As I argued in section I, no substance's states of moving and resting (in contrast to merely being *at rest*) are contiguous but are always separated by at least one instant that belongs to neither of those logically contrary states. This result follows from the premise that moving and resting are both processes. Likewise, whenever the demise of one substance gives rise to a second substance, the existence of the first substance is separated from the existence of the second substance by at least one instant. This result follows from the premise that any substance's existing at t implies its aging at t .

According to Graham Priest, "A cup is both a cup and not a cup the instant it fractures into smithereens."¹⁵ Priest thus anticipates and endorses the instantaneously inconsistent situations described by Nozick. I have shown why we do not need to go that desperate route: there is no last instant at which the cup exists, there is a first instant at which the cup no longer exists, and there is no instant at which the cup is not a cup. If the cup's fracturing gives rise to smithereens, and those smithereens are substances, then at least one instant passes between the cup's existence and theirs, an instant at which neither cup nor smithereens exist but only the individual molecules (and perhaps their mereological sum) that existed, along with the cup, before the cup's demise. As I visualize the process of fracturing, this result seems entirely plausible, even if we cannot hope to *know* just which instant it is at which neither cup nor smithereens exist.

Similarly, when the beta decay of a neutron gives rise to a proton, an electron, and an electron antineutrino, at least one instant must pass between the neutron's existence and theirs. Importantly, "at least one instant" does not imply "some measurable nonzero span of time," so my

¹⁵ Priest, "To Be and Not to Be," *op. cit.*, 266.

claim is not hostage to the possibility of our measuring some very brief interval. When a parent cell divides and gives rise to daughter cells, the parent and daughter cells do not co-exist, and the demise of the former is separated by at least one instant (not necessarily a measurable span of time) from the emergence of the latter. Consider one last example, because it is different: when a tadpole becomes a frog, a single substance—a single organism—undergoes qualitative change from one phase to another, and the organism ages continuously throughout the change. Because it is not substantial change, our answer in this case need not accommodate the existence, and therefore the aging, of two distinct substances.

One might wonder if my analysis of the temporal boundaries and contiguity of substances can answer analogous questions about the *spatial* boundaries and contiguity of substances.¹⁶ Although I am not entirely sure that it can, the following tentative remarks may provide some reason to think so. If we take seriously the unification of space and time into spacetime, then we ought to hold that no point of space exists except at an instant of time (and conversely). In that case, no substance can occupy a first or a last point of space, because it cannot occupy a first or a last instant of time: every substance must have open spatial as well as temporal boundaries.

It follows that no two substances can touch unless they *overlap*, that is, unless they share infinitely many spacetime points: non-overlapping substances must always be separated by at least one spacetime point, although (again) not necessarily by a measurable nonzero spacetime interval. Importantly, the impossibility of substances that touch without overlapping does not make *contact* between substances topologically impossible, because substances can share infinitely many spacetime points even if there cannot be a first (or *n*th, for any integer *n*) spacetime point that they

¹⁶ I thank an anonymous referee from this JOURNAL for asking this question.

share. It also allows a substance to touch its mereological *complement* without overlapping, on the highly plausible assumption that the complement of a substance is never itself a substance.¹⁷

IV. Endurantism

My approach to the problem of the instant of substantial change treats aging as a process that every substance undergoes at each instant of its existence. One might wonder if my approach requires taking other participles, such as “happening” and “unfolding,” to signal processes that every diachronic (that is, non-instantaneous) *event* undergoes at each of its constituent instants, in which case no diachronic event contains a first or a last instant. But if so, then the following problem arises for my approach.

If no substance has a first instant of existence, then the diachronic event (if any) that is the total prehistory of any substance’s existence must have a last instant. Otherwise, an instant must fall between the substance’s total prehistory and its existence, in violation of classical logic. Likewise, if no substance has a last instant of existence, then the diachronic event (if any) that is the total aftermath of the substance’s existence must have a first instant. Otherwise, an instant must fall between the substance’s existence and its total aftermath, again in violation of classical logic. One might try to evade the problem by denying that there are any such events as a substance’s total prehistory or aftermath, but I see no principled reason to deny it. So I must deny that happening, unfolding, transpiring (and so on) are processes that every event undergoes at each of its instants.

¹⁷ Compare Achille Varzi, “Boundaries, Continuity, and Contact,” *Noûs* 31: 1 (1997): 26–58, at 32–36. David S. Oderberg, “Coincidence Under a Sortal,” *Philosophical Review* 105: 2 (1996): 145–71, defends the claim that distinct substances can overlap in spacetime.

Fortunately, there is a principled way of defending that response. For a substance to be aging at an instant t , it seems that the substance must be “wholly present” at t , or at least more than a zero-measure fraction of the substance’s parts must be present at t . Even if the baseball discussed in section I lacks some of the cowhide or stitching it had when it was new, the baseball is still wholly present at each instant at which it exists, or at least the baseball is not missing almost all of its parts at every instant of its existence. By contrast, with any diachronic event, no more than a zero-measure fraction of its parts (namely, exactly one of its instants) exists at any instant; none of its infinitely many other parts do. Intuitively, for anything to be undergoing any process at t , it must not be almost entirely absent at t . Therefore, no diachronic event undergoes any process, including happening or unfolding, at any instant. We must treat as picturesque, rather than literally true, the claim that some diachronic event “is happening now” if “now” denotes an instant. In this way, my approach avoids the problems that would arise if a substance’s prehistory had to lack a last instant or its aftermath had to lack a first instant.

By claiming that substances but not events are wholly present—or, more cautiously, not almost wholly absent—at each instant of their existence, I commit myself to some version of *endurantism* about substances. But distinguishing events, which consist of temporal parts, from substances, which do not, allows a consistent solution to the problem of the instant of substantial change. Furthermore, I see no special resources in the rival view, *perdurantism*, for solving the problem. *Perdurantism* does not answer the question whether my prehistory has a last instantaneous temporal part or whether I have a first instantaneous temporal part, and it blocks at least one consistent way of answering that question.

V. Conclusion

It is noteworthy that solving the problem of the instant of substantial change required no idealization, in contrast to the problems about qualitative change (color and motion) that I discussed earlier. Because a substance's color depends to some extent on the nature of the observer, sorting out changes of color requires idealizations about observers and the conditions of observation. Sorting out motion requires idealizations like those I used in section I. But existence is not in any sense observer-dependent,¹⁸ nor is it a physical concept and thereby hostage to the particular details of physics that characterize our universe. If existence were a physical concept, then the phrase "physically exist" would be redundant, which it is not; ontological physicalism, even if true, is not true by definition. Again, this feature of the problem of substantial change gives the problem a robustness and objectivity that enhance its importance.

I will conclude by heading off two potential misinterpretations of my proposal and by answering a potential objection to it.

First, it is important not to misread the claim that no substance has an initial or final instant of its existence as implying that the substance always has existed or always will exist. The open real-number interval $1 < t < 2$ might represent the timeline of some substance. The interval contains no smallest or largest member, but the interval itself is only finite in its extent: it lacks 1 and any smaller numbers; it lacks 2 and any larger numbers. No instant was my first, and none will be my last, but it does not follow that I am infinitely old or immortal.

¹⁸ Notwithstanding the usual portrayal of Schrödinger's Cat, which is based on a particular, minority position in the interpretation of quantum mechanics, and arguably based on a misreading of even that minority position; see Jan Faye, "Copenhagen Interpretation of Quantum Mechanics," in Edward N. Zalta, ed., *The Stanford Encyclopedia of Philosophy* (Fall 2014 edition), <https://plato.stanford.edu/archives/fall2014/entries/qm-copenhagen/#MisCom>.

Second, my proposal does not imply that any substance that exists at t , and hence is aging at t , existed for any particular nonzero interval starting earlier than t or will continue to exist for any particular nonzero interval ending later than t . For any nonzero interval before or after t , no matter how small, the substance may not have existed, and may not continue to exist, through all of that interval. Aging does not guarantee any particular nonzero lifespan, although it does guarantee some nonzero lifespan or other. Indeed, for those who think that time could have a beginning or an end, a substance's aging at t does not imply that time itself did not begin, or that time itself will not end, arbitrarily close to t . I hasten to add that time's having a beginning or an end does not imply the existence of a first or a last *instant* of time, any more than there being a beginning or an end to my existence requires a first or a last instant of my existence. Time itself could be a finite interval that is open at both ends.¹⁹

Finally, one might object to my account of substantial change precisely because it presupposes the passage of time, an allegedly outmoded idea refuted by the scientifically respectable B-theory of time.²⁰ It is true that my account depends on the premise that every substance is aging at every instant at which it exists, but it is hard to see how that premise conflicts with any scientifically respectable theory of time. On the contrary, the theory of special relativity

¹⁹ Contemporary cosmology may provide some support for this view. See Chris Smeenk "Time in Cosmology," in Heather Dyke and Adrian Bardon, eds., *A Companion to the Philosophy of Time* (Malden, MA: Wiley, 2013), 201–19, at 207–08. Time need not, of course, be a *substance* to be open at both ends. For at least the following reason, broached in section IV above, we should resist concluding that time *must* be open at both ends because time is *in the process of passing* at each of its constituent instants. To be undergoing a process at t , time must be wholly present (or at least not almost wholly absent) at t , and whatever time is, it seems clear that time is not wholly present at any of its instants.

²⁰ Jenann Ismael, however, argues that the B-theory of time does not imply that the passage of time is merely an illusion: "From Physical Time to Human Time," in Yuval Dolev and Michael Roubach, eds., *Cosmological and Psychological Time* (Dordrecht: Springer Publishing, 2016), 107–24.

is often taken to support the B-theory of time, and special relativity famously says that substances can *age at different rates* depending on how the substances move with respect to each other. The theory of general relativity, in turn, says that substances can age at different rates due to the effect of gravity, but (as far as I know) it does not say that the effect of gravity ever allows a substance to exist without aging at all. Both theories, therefore, seem compatible with the key premise of my argument.

Nozick warns us against “overreacting”²¹ to his suggestion that, in order to answer questions about the instant of change, we may have to accept that our world contains some (indeed, bewilderingly many) situations that are contradictory. My argument cautions us not to overreact to the problem of substantial change by flirting with Nozick’s somewhat reluctant acceptance of inconsistency, much less Priest’s enthusiastic embrace of it.²²

Stephen Maitzen
Acadia University

²¹ Nozick, *Invariances*, *op. cit.*, 304.

²² For helpful comments, I thank Niko Strobach, John L. Bell, and two anonymous referees for this JOURNAL.